NOTATION

μ, discharge coefficient; Δp, static pressure gradient on the permeable wall; q_c, dynamic pressure in the channel from which the discharge occurs; q_f, dynamic pressure of entraining flow; ρ, density; c, permeability of wall; d, diameter of holes or width of slit; D, diameter of tube; H, height of channel; Re, Reynolds number.

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ANALYSIS OF THE FLOW OF A NONLINEAR VISCOUS FLUID FOR CYCLIC DEFORMATION

N. G. Bekin, A. A. Lomov, and G. M. Goncharov

UDC 532.135

A hydrodynamic analysis was carried out for the propagation of shear vibrations in a layer of a nonlinear viscous "power-law" fluid.

Theoretical description and development of the methods of calculating and treating the cyclic deformation of polymeric materials are associated with great difficulties. They are caused not only by the necessity of solving nonlinear differential equations, but also of finding the acoustical characteristics of the substance being treated and their connections with its rheological properties. The following scheme of cyclic deformation is considered to be the most general one. A material is considered to be enclosed between two parallel plates, one of which executes harmonic vibrations in its plane (Fig. 1). In this case, from the vibrating plate there propagate shear waves perpendicular to the direction of motion of the plate. If the other plate is fixed, the waves are partly reflected from it, and partly damped. In the general case for a thin layer of fluid, the propagation of the waves can be described by the equation [1]

$$v = C_1 \exp i(-ky + \omega t) + C_2 \exp i(ky + \omega t). \tag{1}$$

In this case it is assumed that the waves excited by the vibrating plate are not distorted in form. This assumption is correct for the low-frequency range of cyclic deformation of highly viscous polymers and can be confirmed by using harmonic analysis of experimentally recorded low-frequency vibrations in a channel filled with a "power-law" material [2].

The boundary conditions in the present case have the form

$$v = v_0 \exp i\omega t \text{ for } y = 0, \; v = 0 \text{ for } y = h. \tag{2}$$

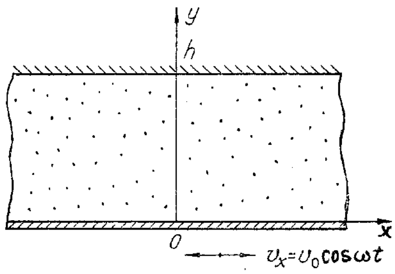

Fig. 1. Diagram of the deformation process.

Yaroslavl Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 1, pp. 58-62, July, 1982. Original article submitted May 14, 1981.

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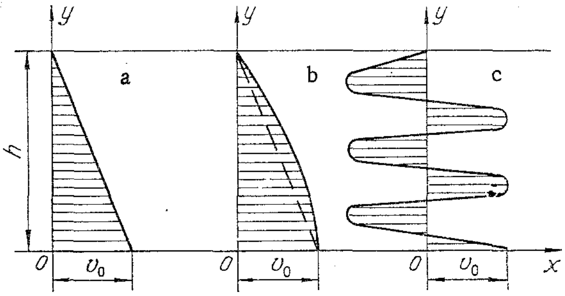

Fig. 2. Variation of velocity profile of material for an increase in the frequency of vibration of the plate ($n = 0.3$; $\mu = 1\cdot 10^5$ nsec/m^2; $A = 1\cdot 10^{-3}$ m; $\rho = 10^3$ kg/m^3; $h = 2\cdot 10^{-2}$ m): a) $\omega < 2\cdot 10^4$ sec^{-1}; b) $\omega \approx 2\cdot 10^4$ sec^{-1}; c) $\omega = 5\cdot 10^5$ sec^{-1}.

The complex form of writing Eqs. (1) and (2) is convenient for carrying out calculations: however, in what follows, only the real part is necessary.

Using the boundary conditions (2) we obtain C_1 and C_2, and after transformation we obtain the velocity distribution of the material over the height of the channel

$$v = v_0\left[\cos(ky - \omega t) - \frac{\sin ky}{\sin kh}\cos(kh - \omega t)\right]. \tag{3}$$

In the expression obtained, a quantity that is still unknown is the propagation constant k (for an inelastic fluid, equal to the wave number), which in the case of a Newtonian fluid equals $\sqrt{\rho\omega/2\mu}$ [3].

The aim of the present study is to find the values of the quantity k and the velocity distribution for a power-law fluid included between plates, the rheological equation of which in the case being considered has the form [4]

$$\tau = \mu\left|\frac{\partial v}{\partial y}\right|^n \operatorname{sign}\frac{\partial v}{\partial y}. \tag{4}$$

Knowledge of the propagation constant k for a power-law fluid and of the specific parameters of cyclic deformation enables us to estimate the wavelength λ of the wave propagating in the fluid, the damping of waves, and the velocity profile of the wave over the height of the channel, and also enables us to estimate the inertial forces that arise in a fluid under cyclic deformation.

In the present case the process of propagation of shear vibrations in a layer of a nonlinear viscous, power-law fluid can be described by the equation of motion with the use of (4) and (2). Assuming the process to be unidirectional, we can represent the equation of motion with substitution of the rheological equation in it in the form

$$\rho\frac{\partial v}{\partial t} = \mu n\left|\frac{\partial v}{\partial y}\right|^{n-1}\frac{\partial^2 v}{\partial y^2}. \tag{5}$$

To find the velocity profile of the wave between the plates we integrate (5) with boundary conditions (2), and we approximate it by expression (3) with subsequent determination of k. For $n = 1$ (a Newtonian fluid), Eq. (5) becomes linear, and in this case there exists an analytic solution [5].

We apply the Galerkin method used in [6] for solving the problem of the propagation of transverse vibrations in an unbounded medium, for which from (5) we form the operator

$$L^n(v) \equiv \rho\frac{\partial v}{\partial t} - \mu n\left|\frac{\partial v}{\partial y}\right|^{n-1}\frac{\partial^2 v}{\partial y^2} = 0. \tag{6}$$

In view of the fact that perturbation (because $n \neq 1$) occurs over the variable y, and not over t, we can assume that the periodicity over t in the perturbed problem is preserved. We find the solution of (5), (2) in the form

$$v_s(y, t) = v_0\left[\left(\sum_{i=1}^{s} \alpha_i y^i\right)\cos\omega t + \left(\sum_{i=1}^{s} \beta_i y^i\right)\sin\omega t\right], \tag{7}$$

i.e., we construct a sequence of solutions $v_s(y, t)$.

As was noted above, we neglect the presence of harmonics above the fundamental in the wave, although their effect on the wave profile for essentially pseudoplastic materials ($n = 0.2$–0.4) becomes more significant in comparison with a Newtonian fluid.

It is easy to obtain requirements for the coefficients α_i and β_i, necessary for satisfying the boundary conditions, successively substituting (2) into (7):

$$\left.\begin{array}{l}\alpha_0 = 1\\ \beta_0 = 0\end{array}\right\}, \quad \left.\begin{array}{l}\sum_{i=1}^{s} \alpha_i h^i = 0\\ \sum_{i=1}^{s} \beta_i h^i = 0\end{array}\right\}. \tag{8}$$

Then, following the Galerkin method, we introduce a system of functions of the form

$$\begin{gathered} H_j(y, t) = y^j \cos\omega t,\ t \in \left[0, \frac{2\pi}{\omega}\right],\ y \in [0, h],\\ G_j(y, t) = y^j \sin\omega t,\ j = 0, 1, \ldots, s, \end{gathered} \tag{9}$$

and the scalar product

$$(a(y, t), b(y, t)) \equiv \int_0^h \int_0^{2\pi/\omega} a(y, t)\cdot b(y, t)\,dt\,dy, \tag{10}$$

where by virtue of the periodicity with respect to t we use a time interval from 0 to $2\pi/\omega$.

Thus, we can construct a Galerkin system with basis (9) and relation (8):

$$\left.\begin{array}{l}(L^n(v_s), H_j) = 0,\\ (L^n(v_s), G_j) = 0,\end{array}\right\} j = 1, 2, \ldots, s-1. \tag{11}$$

Substituting (7) into (11) and satisfying the necessary transformations (which have been dropped here because of their complexity), we obtain expressions for the scalar products (11), which in the system with (8) make it possible to determine α_i and $\beta_i (i = 1, 2, \ldots, s-1)$:

$$\begin{gathered} (L^n(v_s), H_j) = \rho v_0 \pi \sum_{i=1}^{s} \beta_i \frac{h^{i+j+1}}{i+j+1} - n\mu v_0^n \sum_{i=2}^{s} i(i-1) \int_0^h \int_0^{2\pi/\omega} y^{i+j+2} r(t, y) f_1^i(t)\,dt\,dy,\\ (L^n(v_s), G_j) = -\rho v_0 \pi h^{j+1}\left[\sum_{i=1}^{s} \alpha_i \frac{h^i}{j+i+1} + \frac{1}{j+1}\right] - n\mu v_0^n \sum_{i=2}^{s} i(i-1) \int_0^h \int_0^{2\pi/\omega} y^{i+j+2} r(t, y) f_2^i(t)\,dt\,dy, \end{gathered} \tag{12}$$

where

$$\begin{gathered} f_1^i(t) = \alpha_i \cos^2\omega t + \beta_i \cos\omega t \sin\omega t;\ f_2^i(t) = \alpha_i \cos\omega t \sin\omega t + \beta_i \sin^2\omega t;\\ r(t, y) = \left|\sum_{i=1}^{s} i y^{i-1} (\alpha_i \cos\omega t + \beta_i \sin\omega t\right|^{n-1}. \end{gathered} \tag{13}$$

The system (3), (7), (8), (11), (12) was solved on a BÉSM-6 computer, as a result of which we found dependences of the velocity of the fluid between the plates v, the propagation constant k, the wavelength $\lambda = 2\pi/k$ from the properties of the fluid (μ, n, ρ) and the parameters of cyclic deformation (v_0, ω). We obtained that k increases with increasing values of μ, n, ρ, ω, and v_0; however, the index of flow of the material proves to have the strongest effect on k.

Calculations were also carried out for specific fluids, such as rubber mixtures based on SKN (a code mark for butadiene-acrylonitrile synthetic rubber), and as parameters of cyclic deformation we took the frequently encountered values of frequency and amplitude $A = v_0/\omega$ of the vibrational action [2, 7].

It was established that for distance between the plates not exceeding 20 mm ($h \leq 20$ mm was chosen on the basis of a real scheme of deformation in the space between the vibrating wedge and roller of the calendar [7]), and of polymeric materials, such as rubber mixtures (having properties of nonlinear viscous fluids), the profile of velocity between the plates remains linear in the range of frequencies $\omega < 2 \cdot 10^4$ sec^{-1} for $A < 3$. This is explained by the value of the wavelength $\lambda \gg h$, propagating in the fluid. Therefore, two important conclusions follow.

1. In the indicated range of cyclic deformation of nonlinear viscous fluids (rubbers, rubber mixtures) for development of methods of calculation and treatment of indicated materials we can neglect the inertial forces of the material, since, assuming $\rho(\partial v/\partial t) = 0$ in Eq. (5) and solving this together with (2), we obtain a linear velocity distribution

$$v = v_0 \cos \omega t \left(1 - \frac{y}{h}\right), \tag{14}$$

which with accuracy needed for engineering calculations describes the behavior of the material for vibratory action with respect to the scheme being investigated.

2. Operating with the linearity of the velocity profile we can find the optimal parameters of the cyclic deformation, for which we attain the highest intensification of the processes of treatment of polymeric materials [2], since in this case the tangential stresses τ distributed with respect to the channel height have a constant value for given value of time. This can easily be verified, after substituting (14) into (4).

For higher frequencies $\omega > 2 \cdot 10^4$ sec^{-1} the velocity profile begins to bend, i.e., λ becomes comparable with h, and with further increase, the frequency λ becomes much less than h. In the last case it is necessary to take account of the damping of the vibrations in a layer of fluid and the distortion of their forms. Figure 2 represents the velocity profiles of the fluid between plates for various values of frequency of vibrations of one of them.

NOTATION

v, velocity of the material in the direction of the x axis; ω, angular frequency; k, propagation constant; t, time; h, distance between the plates; ρ, density of the fluid; μ, degree of consistency; n, index of the flow; τ, stress tensor components; λ, wavelength; α_i, β_i, unknown coefficients of the series; v_0, peak value of the velocity of the plate; and A, amplitude of the motion of the plate.

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